Warwick EuroConference (July 1996)

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Abstract

This file, provided as bibliographical information for the benefit of contributors, contains the table of contents and editors' foreword from the proceedings of the July 1996 Warwick EuroConference on Algebraic Geometry.

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Contents:

Foreword

The volume contains a selection of seventeen survey and research articles from the July 1996 Warwick European algebraic geometry conference. These papers give a lively picture of current research trends in algebraic geometry, and between them cover many of the outstanding hot topics in the modern subject. Several of the papers are expository accounts of substantial new areas of advance in mathematics, carefully written to be accessible to the general reader. The book will be of interest to a wide range of students and nonexperts in different areas of mathematics, geometry and physics, and is required reading for all specialists in algebraic geometry.

The European algebraic geometry conference was one of the climactic events of the 1995–96 EPSRC Warwick algebraic geometry symposium, and turned out to be one of the major algebraic geometry events of the 1990s. The scientific committee consisted of A. Beauville (Paris), F. Catanese (Pisa), K. Hulek (Hannover) and C. Peters (Grenoble) representing AGE (Algebraic Geometry in Europe, an EU HCM–TMR network) and N.J. Hitchin (Oxford), J.D.S. Jones and M. Reid (Warwick) representing Warwick and British mathematics. The conference attracted 178 participants from 22 countries and featured 33 lectures from a star-studded cast of speakers, including most of the authors represented in this volume.

The expository papers Five of the articles are expository in intention: among these a beautiful short exposition by Paranjape of the new and very simple approach to the resolution of singularities; a detailed essay by Ito and Nakamura on the ubiquitous ADE classification, centred around simple surface singularities; a discussion by Morrison of the new special Lagrangian approach giving geometric foundations to mirror symmetry; and two deep and informative survey articles by Behrend and Siebert on Gromov–Witten

invariants, treated from the contrasting viewpoints of algebraic and symplectic geometry.

Some main overall topics Many of the papers in this volume group around a small number of main research topics. Gromov–Witten invariants was one of the main new breakthroughs in geometry in the 1990s; they can be developed from several different starting points in symplectic or algebraic geometry. The survey of Siebert covers the analytic background to the symplectic point of view, and outlines the proof that the two approaches define the same invariants. Behrend's paper explains the approach in algebraic geometry to the invariants via moduli stacks and the virtual fundamental class, which essentially amounts to a very sophisticated way of doing intersection theory calculations. The papers by Paoletti and Wilson give parallel applications of Gromov–Witten invariants to higher dimensional varieties: Wilson's paper determines the Gromov–Witten invariants that arise from extremal rays of the Mori cone of Calabi–Yau 3-folds, whereas Paoletti proves that Mori extremal rays have nonzero associated Gromov–Witten invariants in many higher dimensional cases. The upshot is that extremal rays arising in algebraic geometry are in fact in many cases invariant in the wider symplectic and topological setting.

Another area of recent spectacular progress in geometry and theoretical physics is Calabi–Yau 3-folds and mirror symmetry. This was another major theme of the EuroConference that is well represented in this volume. The paper by Voisin, which is an extraordinary computational tour-de-force, proves the generic Torelli theorem for the most classical of all Calabi–Yau 3-folds, the quintic hypersurface in $\mathbb{P}^4$. The survey by Morrison explains, among other things, the Strominger–Yau–Zaslow special Lagrangian interpretation of mirror symmetry. Beauville's paper gives the first known construction of a Calabi–Yau 3-fold having the quaternion group of order 8 as its fundamental group. The paper by Batyrev proves that the Betti numbers of a Calabi–Yau 3-fold are birationally invariant, using the methods of p-adic integration and the Weil conjectures; the idea of the paper is quite startling at first sight (and not much less so at second sight), but it is an early precursor of Kontsevich's idea of motivic integration, as worked out in papers of Denef and Loeser. Several other papers in this volume (those of Ito and Nakamura, Mukai, Shioda and Wilson) are implicitly or explicitly related to Calabi–Yau 3-folds in one way or another.

Other topics The remaining papers, while not necessarily strictly related in subject matter, include some remarkable achievements that illustrate the breadth and depth of current research in algebraic geometry. Shioda extends his well-known results on the Mordell–Weil lattices of elliptic surfaces

to higher genus fibrations, in a paper that will undoubtedly have substantial repercussions in areas as diverse as number theory, classification of surfaces, lattice theory and singularity theory. Faber continues his study of tautological classes on the moduli space of curves and Abelian varieties, and gives an algorithmic treatment of their intersection numbers, that parallels in many respects the Schubert calculus; he obtains the best currently known partial results determining the class of the Schottky locus. Gizatullin initiates a fascinating study of representations of the Cremona group of the plane by birational transformations of spaces of plane curves. Eyssidieux gives a study, in terms of Gromov's Kähler hyperbolicity, of universal inequalities holding between the Chern classes of vector bundles over Hermitian symmetric spaces of noncompact type admitting a variation of Hodge structures. Küchle and Steffens' paper contains new twists on the idea of Seshadri constants, a notion of local ampleness arising in recent attempts on the Fujita conjecture; they use in particular an ingenious scaling trick to provide improved criteria for the very ampleness of adjoint line bundles.

Manetti's paper continues his long-term study of surfaces of general type constructed as iterated double covers of $\mathbb{P}^2$. He obtains many constructions of families of surfaces, and proves that these give complete connected components of their moduli spaces, provided that certain naturally occuring degenerations of the double covers are included. This idea is used here to establish a bigger-than-polynomial lower bound on the growth of the number of connected components of moduli spaces. In more recent work, he has extended these ideas in a spectacular way to exhibit the first examples of algebraic surfaces that are proved to be diffeomorphic but not deformation equivalent.

The Fourier–Mukai transform is now firmly established as one of the most important new devices in algebraic geometry. The idea, roughly speaking, is that a sufficiently good moduli family of vector bundles (say) on a variety X induces a correspondence between X and the moduli space $\widehat{X}$. In favourable cases, this correspondence gives an equivalence of categories between coherent sheaves on X and on $\widehat{X}$ (more precisely, between their derived categories). The model for this theory is provided by the case originally treated by Mukai, when X is an Abelian variety and $\widehat{X}$ its dual; Mukai named the transform by analogy with the classical Fourier transform, which takes functions on a real vector space to functions on its dual. It is believed that, in addition to its many fruitful applications in algebraic geometry proper, this correspondence and its generalisations to other categories of geometry will eventually provide the language for mathematical interpretations of the various "dualities" invented by the physicists, for example, between special Lagrangian geometry on a Calabi–Yau 3-fold and coherent algebraic geometry on its mirror partner (which, as described in Morrison's article, is conjecturally a fine moduli space for special Lagrangian tori). Mukai's magic paper in this volume

presents a Fourier–Mukai transform for K3 surfaces, in terms of moduli of semi-rigid sheaves; under some minor numerical assumptions, he establishes the existence of a dual K3 surface, the fact that the Fourier–Mukai transform is an equivalence of derived categories, and the biduality result in appropriate cases.

The paper of Ito and Nakamura is the longest in the volume; it combines a detailed and wide-ranging expository essay on the ADE classification with an algebraic treatment of the McKay correspondence for the Kleinian quotient singularities $\mathbb{C}^2/G$ in terms of the G-orbit Hilbert scheme. The contents of their expository section will probably come as a surprise to algebraic geometers, since alongside traditional aspects of simple singularities and their ADE homologues in algebraic groups and representation of quivers, they lay particular emphasis on partition functions in conformal field theory with modular invariance under $\mathrm{SL}(2,\mathbb{Z})$ and on II_1 factors in von Neumann algebras. Their study of the G-Hilbert scheme makes explicit for the first time many aspects of the McKay correspondence relating the exceptional locus of the Kleinian quotient singularities $\mathbb{C}^2/G$ with the irreducible representations of G; for example, the way in which the points of the minimal resolution can be viewed as defined by polynomial equations in the character spaces of the corresponding irreducible representations, or the significance in algebraic terms of tensoring with the given representation of G. Ito and Nakamura and their coworkers are currently involved in generalising many aspects of the G-orbit Hilbert scheme approach to the resolution of Gorenstein quotient singularities and the McKay correspondence to finite subgroups of $\mathrm{SL}(3,\mathbb{C})$, and this paper serves as a model for what one hopes to achieve.

Thanks to all our sponsors The principal financial support for the EuroConference was a grant of ECU40,000 from EU TMR (Transfer and Mobility of Researchers), contract number ERBFMMACT 950029; we are very grateful for this support, without which the conference could not have taken place. The main funding for the 1995–96 Warwick algebraic geometry symposium was provided by British EPSRC (Engineering and physical sciences research council). Naturally enough, the symposium was one of the principal activities of the Warwick group of AGE (European Union HCM project Algebraic Geometry in Europe, Contract number ERBCHRXCT 940557), and financial support from Warwick AGE and the other groups of AGE was a crucial element in the success of the symposium and the EuroConference. We also benefitted from two visiting fellowships for Nakamura and Klyachko from the Royal Society (the UK Academy of Science). Many other participants were covered by their own research grants.

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Klaus Hulek and Miles Reid, November 1998